Applications of Differential Equations Equation Sheet

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# Second Order Linear Homogeneous Constant Coefficient ODEs

* + - Auxiliary equation
    - Solution
      * If
      * If
      * If

# Second Order Linear Homogeneous ODEs

* + General form
  + Standard form IVP

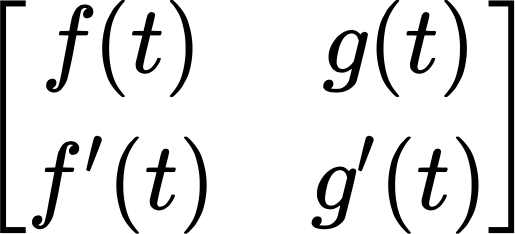
## Uniqueness and Existence Theorem

* + - * Assuming and are continuous on and is also on
      * Then the standard form IVP will always have a unique solution
      * Now we can just look at and on some interval around called and determine if the ODE is solvable without having to solve it

# Superposition Theorem

* + Any linear combination of solutions to an ODE is also a solution to the ODE

# Wronskian

* + 
  + If then and are linearly independent
  + Else then and are linearly dependent and vary by a constant multiple

## Abel’s theorem

* + - This means you can compute the wronskian without knowing solutions to the equation

# Reduction of Order

* + For and a non trivial solution

# Euler's equation

* + Solve any equation of the form
    - Use the substitution and
  + With the substitution, becomes

# Variation of Parameters

# Oscillation

* + Normal Form of a differential equation
  + Any standard form equation using:

## Oscillation Criteria Theorem

* + - * If then the equation is oscillatory

## Sturm's Separation Theorem

* + - If the normal form has oscillatory solutions and . If and on then between the interval there must be a point such that

## Sturm's Comparison Theorem

* + - Consider the DE’s and
    - Suppose is a known solution with and on
    - If on , Then if has one zero on there will be another zero where c is on

# Nonlinear Differential Equations

* + General form for 2nd Order:
    - Consider the form

## Solvable Case 1:

* + - Missing the term
    - Method: let and
      * Solve for then

## Solvable Case 2:

* + - Missing the term
    - Method: let and
      * Then solve resulting equation
      * Do not cancel out if it appears on both sides

## Solvable Case 3: and polynomial degree of each term is all the same

* + - Example:
    - Method: let , ,

# Classification of points

* + For the 2nd order DE of form:
  + If is an ordinary point
    - Power series solution applies
  + is a singular point if or are undefined
    - is an regular singular point if
      * If and exist
    - Irregular singular point if they do not exist
    - Frobenius series solution for regular singular points

# Bessel's equation

# Bessel’s function

* + When is an integer
* Linear systems of DE’s

## Eigen-stuff

* + - Find and
    - If are real
      * Unbounded solutions
    - If are complex
      * Periodic solutions

## Hamiltonian

* + - Equilibrium point
      * and
    - For linear systems: